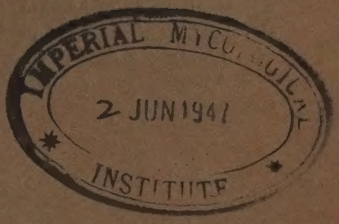
COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1943



VOLUME XXXIX

№ 5

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
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1943

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Édition de l'Académie des Sciences de l'URSS
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1943. Volume XXXIX, № 5

MATHEMATICS

CLASSIFICATION OF ORDERINGS OF LINEAR SYSTEMS

By D. WASSILKOFF

(Communicated by S. L. Soboleff, Member of the Academy, 11. I. 1943)

In the present note we investigate some properties of various orderings of a fixed linear system, between which a relation of order is defined in a quite natural way. By ordering we understand, in general, *partial* ordering.

1. Orderings of Arbitrary Sets. Let $M=\{x,y,\dots\}$ be any set. Denote by Ω an arbitrary ordering of M, i. e. a law that converts M into a partially ordered set (1). To avoid ambiguity, in the following the symbol $[\Omega]$ will indicate that the expression preceding it is to be understood *in the sense of the ordering* Ω (for instance, $x<y\,[\Omega]$).

Consider the orderings Ω' and Ω'' of the set M. We shall say that Ω' is included in Ω'' (in symbols: $\Omega'\leqslant\Omega''$ or $\Omega''\geqslant\Omega'$) if $x<y\,[\Omega']$ implies $x<y\,[\Omega'']$.

The totality $\overline{\mathfrak{M}}=\{\Omega\}$ of all orderings of M becomes thus ordered itself. It is appropriate to introduce the void ordering, in the sense of which any two elements $x\in M$, $y\in M$ are incomparable (in symbols: $x\parallel y$). The void ordering may be considered as included in every $\Omega\in\overline{\mathfrak{M}}$.

Let $\{\Omega^\alpha\}$ $(\alpha\in A)$ be an arbitrary system of orderings. It is easily seen that $\overline{\mathfrak{M}}$ contains the meet (2) of the system $\{\Omega^\alpha\}$ denoted by $\Omega=\prod_\alpha \Omega^\alpha$ (that may casually be void). Ω can be defined as follows: $x<y\,[\Omega]$ *if* $x<y\,[\Omega^\alpha]$ *for all* $\alpha\in A$.

If the system $\{\Omega^\alpha\}$ is bounded above, then there exists the join (2) of $\Omega^\alpha(\alpha\in A)$ in $\overline{\mathfrak{M}}$.

Theorem 1. *The totality* $\overline{\mathfrak{M}}$ *of all orderings of an arbitrary set* M *forms a closed ordered set* (4) *possessing the least element.*

We call the ordering Ω irreducible if Ω cannot be represented as the meet of a system of orderings different from Ω.

We shall say that Ω is a strict ordering if any two elements $x\in M$, $y\in M$ are comparable $[\Omega]$, i. e. either $x\leqslant y\,[\Omega]$ or $x>y\,[\Omega]$ takes place.

Theorem 2. *An ordering* $\Omega\in\overline{\mathfrak{M}}$ *is irreducible if and only if* Ω *is a strict ordering. Any* $\Omega\in\overline{\mathfrak{M}}$ *can be represented as the meet of a system of strict orderings, whose cardinal number does not exceed that of* M.

Such a representation is in general not unique.

2. Orderings of Linear Systems. Let $E=\{x,y,\dots\}$ be a linear system, i. e. an Abelian group with real numbers as operators. We shall consider various orderings of E, but now we shall restrict ourselves to orderings of E, satisfying the following axioms:

Axiom I. *If* $x<y$, *then* $x+z<y+z$ *for any* $z\in E$.

Axiom II. *If $x < y$ and λ is any positive number, then $\lambda x < \lambda y$.*

Such orderings will be called linear orderings. Denote by $\mathfrak{E}$ the set of linear orderings of the linear system E. The void ordering may be included in $\mathfrak{E}$. $\mathfrak{E}$, being a subset of the set $\overline{\mathfrak{E}}$ of all orderings of E, is an ordered set.

Moreover, we have

Theorem 3. *$\mathfrak{E}$ is a closed subset of the ordered set $\overline{\mathfrak{E}}$.*

It is easy to see that $\mathfrak{E}$ *is isomorphic with the naturally ordered system of conic sets C in E* ([3]).

As to the reducibility of linear orderings in the set $\mathfrak{E}$, we have the following theorem, analogous to Theorem 2.

Theorem 4. *A linear ordering Ω is irreducible in $\mathfrak{E}$ if and only if Ω is a strict linear ordering. Every $\Omega \in \mathfrak{E}$ can be represented as the meet of a system of strict linear orderings, whose cardinal number does not exceed that of E.*

Let R_x (for fixed $x \in E$) denote the set $\{\lambda x;\ 0 \leqslant \lambda < +\infty\}$. We shall now consider linear orderings satisfying some of the following axioms.

Axiom R. *If for an element $x \in E$ the set R_x is bounded above, being not bounded below, then $x < 0$.*

Axiom H*. *For any $x > 0$ the set R_x is not bounded above.*

Axiom H. *If for an element $x \in E$ R_x is bounded above and below, then $x = 0$.*

We shall simply say that the ordering Ω is, for instance, Ω_H (or $\Omega_{H,R}$) if Ω satisfies Axiom H (resp. Axioms H and R). If Ω is Ω_R, then we shall say that Ω is a regular ordering. The ordering Ω_H (Ω_{H^*} that is not Ω_H) will be called homogeneous (resp. weakly homogeneous). Homogeneous (weakly homogeneous) regular orderings will be called Archimedean (resp. weakly Archimedean). It is easy to see that an ordering satisfying Axioms R and H* and Archimedean orderings may be characterized by Axioms A° and A of my paper ([4]), respectively.

Theorem 5. *Among all regular orderings including an arbitrary fixed $\Omega \in \mathfrak{E}$ there exists the minimal ordering Ω', included in any regular ordering that includes Ω.*

The ordering Ω' will be called the regular hull over Ω.

3. Archimedean and Homogeneous Orderings. Archimedean orderings, as well as closed orderings, with which the former are closely related [([4]), Theorem 3], constitute the most important classes of linear orderings. The following theorem establishes the properties of Archimedean orderings with respect to the lattice operations in $\mathfrak{E}$.

Theorem 6. *The meet of any system and the sum of a finite number of Archimedean orderings (if the latter exists) are Archimedean orderings.*

The following theorem reveals the connexion between the Archimedean orderings and the wider class of homogeneous orderings.

Theorem 7. *The linear ordering Ω is homogeneous if and only if there exists an Archimedean ordering $\Omega' \geqslant \Omega$. The regular hull over the homogeneous ordering Ω is the least Archimedean ordering including Ω.*

From this theorem follow

Corollary 1. *If $\Omega' < \Omega$ and Ω is Ω_H, then Ω' is Ω'_H.*

Corollary 2. *If all Ω^α ($\alpha \in A$) are homogeneous, then $\prod_\alpha \Omega^\alpha$ is likewise homogeneous.*

The Archimedean and homogeneous orderings in the statement of Theorem 7 may be replaced by weakly Archimedean and weakly homogeneous orderings, respectively.

4. Adjoint Orderings. Let F denote the system of all linear functions on E, i. e. such real functions $f(x)$ defined on E that $f(\lambda x + \mu y) = \lambda f(x) + \mu f(y)$ for any elements $x \in E, y \in E$ and numbers λ, μ. The totality of all linear orderings of F will be denoted by $\mathfrak{F}$.

An arbitrary $\Omega \in \mathfrak{E}$ ($\Omega' \in \mathfrak{F}$) generates an ordering $\overline{\Omega} \in \mathfrak{F}$ ($\Omega' \in \mathfrak{E}$) as follows: $f_1 < f_2 [\overline{\Omega'}]$ if $f_1(x) \leqslant f_2(x)$ for all $x \geqslant 0 [\Omega]$ and $f_1(x_0) < f_2(x_0)$ for

at least one $x_0 > 0[\Omega]$ (respectively: $x_1 < x_2[\Omega']$ if $f(x_1) \leqslant f(x_2)$ for al $f \geqslant 0[\overline{\Omega}']$ and $f_0(x_1) < f_0(x_2)$ for at least one $f_0 > 0[\overline{\Omega}']$). The ordering $\overline{\Omega}$ (resp. Ω') thus defined will be called adjoint to Ω (resp. to $\overline{\Omega}'$).

$\Omega \in \mathfrak{E}$ will be called proper ordering if for any $x \in E$, $y \in E$ there exists $z \in E$ such that $z \geqslant x$, $z \geqslant y$ $[\Omega]$.

Lemma. *$\Omega \in \mathfrak{E}$ is a proper ordering if and only if $f = 0$ is the unique linear function that vanishes identically on the set of elements $x > 0[\Omega]$.*

This lemma justifies the following definition: the ordering $\overline{\Omega} \in \mathfrak{F}$ is called weakly proper if $x = 0$ is the unique element of E such that $f(x) = 0$ for all $f > 0$ $[\overline{\Omega}]$.

According to [4] (Theorem 2 and the remark) the following theorems are true.

Theorem 8. *For any $\Omega \in \mathfrak{E}$ the adjoint ordering $\overline{\Omega}$ is $\overline{\Omega}_{R,H^*}$. Ω is Archimedean if and only if Ω is proper.*

Theorem 9. *For any $\overline{\Omega} \in \mathfrak{F}$ the adjoint ordering Ω is $\Omega_{R.H^*}$. Ω is Archimedean if and only if $\overline{\Omega}$ is weakly proper.*

It is possible to prove that the theorem inverse to Theorem 9 is likewise true: *every $\Omega \in \mathfrak{E}$ satisfying Axioms* R *and* H* *is adjoint to an $\Omega \in \mathfrak{F}$.*

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Received
5.I.1943.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

MATHEMATICS

CLASSIFICATION OF CERTAIN SYSTEMS OF FUNCTIONS

By A. T. TALDYKIN

(Communicated by S. L. Soboleff, Member of the Academy, 11. I. 1943)

Examining a system of functions with norms from $L^2(a, b)$

$$\{\varphi_i(x)\}, \tag{1}$$

let us form its main matrix

$$\Phi = \| \varphi_{ik} \|, \quad \varphi_{ik} = \int_a^b \varphi_i(x)\, \bar{\varphi}_k(x)\, dx \quad (i, k = 1,2,\ldots). \tag{2}$$

This Hermitian matrix, semi-bounded below, represents a linear spectrum which is, generally speaking, unbounded.

The bounded operator inverse to the operator $\Phi - \lambda E$ (E, unit-matrix), i. e.

$$R_\lambda = (\Phi - \lambda E)^{-1}, \tag{3}$$

is called the resolvent of the operator of the matrix (2).

In this paper such systems of functions (1) are considered whose matrices (2) possess a resolvent satisfying the functional equation of Hilbert:

$$R_\mu - R_\lambda = (\mu - \lambda)\, R_\mu R_\lambda .$$

To these matrices of systems of functions belong, in particular, the bounded matrices and the Q-matrices, i. e. such as satisfy the condition that

$$\sum_{k=1}^{\infty} | \varphi_{ik} |^2 < +\infty \quad (i = 1, 2, \ldots).$$

The values of λ for which there exists the resolvent (3) of the matrix (2) are called regular for this matrix. The points that are not regular form the spectrum of the matrix (2). For the matrices of function systems this spectrum lies on the non-negative part of the real axis, and in our case presents a closed set [1].

With regard to the number λ_0 the three following cases are possible.

1) λ_0 is a regular value of the resolvent;

2) λ_0 is a proper value of the matrix (2);

3) λ_0 is a point of the spectrum, but in $l^{(2)}$ there is no element $r \neq 0$ such that $\Phi_r = \lambda_0 r$.

We shall now especially examine the case of $\lambda_0 = 0$.

1. Zero does not belong to the spectrum of the matrix (2) if and only if there exists at least one system of functions in $L^2(a, b)$:

$$\{\varphi^i(x)\} \tag{1'}$$

with uniformly bounded norms $N(\varphi^i) = \varphi^{ii} = \int_a^b | \varphi^i(x) |^2 dx < M, \quad (i = 1, 2, \ldots)$

which is a biorthogonal system with respect to (1):

$$\int_a^b \varphi^i(x)\,\bar{\varphi}_k(x)\,dx = \delta_k^i, \quad (i, k = 1, 2, \ldots).$$

A system of functions possessing a biorthogonal is uniformly minimal, i. e. such that not a single function can be discarded from it without decreasing its space. Conversely, a uniformly minimal system has always a biorthogonal (2).

The system

$$\left\{ \varphi^i(x) = \frac{\varphi_i(x) - \hat{\varphi}_i(x)}{\hat{\varphi}_{ii} - \hat{\varphi}_{ii}} \right\} \tag{4}$$

[where $\hat{\varphi}_i(x)$ is the projection of the function $\varphi_i(x)$ onto the space of the remaining functions $\varphi_i(x), , \ldots, \varphi_{i-1}(x), \varphi_{i+1}(x), \ldots$ of the system (1)], biorthogonal with respect to the system (1) and belonging to its space, is called adjoint to (1). With $\lambda = 0$ the value R_λ of the resolvent (3) is the main matrix of system (4) which is the sole system adjoint to (1). The system of functions (1) enjoying these properties has been said to be uniformly minimal (3) or a system with strong minimality.

2. Zero is a proper value of the matrix (2) if and only if the system (1) is not minimal.

The condition is necessary. Suppose the system (1) to be not minimal. In case it is a linearly dependent system, the necessity of the condition is obvious. Let the system (1) be linearly independent and suppose $\varphi_k(x)$, for instance, to belong to the space of the other functions of the system. Then

$$\lim_{n\to\infty} \varphi_n^{kk} = \lim_{n\to\infty} \frac{D_n^{kk}}{D_n} = +\infty$$

where D_n is the determinant of the matrix formed from the elements located at the intersection of n first lines and columns of the matrix (2); D_n^{kk} is the algebraic complement of the element φ_{kk} in this determinant.

This indicates that there is no reciprocal to the operator Φ and, consequently, the mapping $r' = \Phi r$ of the region Ω upon that of values of the operator Φ is not one-to-one. If, on the other hand, the point under investigation, viz. the zero point, is no proper value of the operator Φ, the mapping is one-to-one (1). This shows that in this case zero is the proper value of the matrix (2).

The condition is sufficient. Let zero be the proper value of the matrix (2). Then in Ω there will be found a normalized element $r(x_1, x_2, \ldots)$ satisfying the equation $\Phi r = 0$. This implies

$$(\Phi r, r) = \sum_{\alpha=1}^{\infty}\sum_{\beta=1}^{\infty} \varphi_{\alpha\beta}\, x_\alpha\, \bar{x}_\beta = \int_a^b \left| \sum_{\alpha=1}^{\infty} \bar{x}_\alpha\, \varphi_\alpha(x) \right|^2 dx = 0$$

from which we have $\sum_{\alpha=1}^{\infty} \bar{x}_\alpha \varphi_\alpha(x) \sim 0$ (equivalent to zero) and since all $\bar{x}_\alpha$ do not become zero, at least one function of the system (1) may be expressed through the other, which proves the non-minimality of the system.

3. The zero value belongs to the spectrum of the matrix (2), but is not its proper value if and only if in $L^2(a, b)$ there exists a system of functions (1'), biorthogonal to (1) and having the norms of its functions unbounded in totality.

This is the case when for the operator Φ there exists an unbounded inverse operator Φ^{-1} such that for every r from Ω and for every r' from the determination domain of Φ^{-1} there holds $\Phi^{-1}\Phi r = r$, $\Phi\Phi^{-1} r' = r'$.

In the case considered the norms of functions (4) tend to infinity when $i \to \infty$.

The system (1) enjoying these properties we shall call a weakly minimal system, or a system with weak minimality.

Turning now to the systems of functions (1) whose matrices (2) have the resolvents not necessarily satisfying the functional equation of Hilbert, we likewise shall call them uniformly minimal, weakly minimal or non-minimal, according as the matrix (2) is matched by a reciprocal bounded matrix (2), by a reciprocal unbounded one or by no reciprocal matrix at all.

Thus, all systems of functions fall into three classes:

1) Uniformly minimal systems or systems with strong minimality.

2) Non-minimal systems.

3) Weakly minimal systems or systems with weak minimality.

In studying uniformly minimal systems we subdivided them into three subclasses:

a) Normal systems of functions whose matrix (2) is bounded,

b) Regular systems of functions whose matrix (2) is a Q-matrix.

c) Non-regular systems of functions which are all the remaining uniformly minimal systems.

Some properties of the above systems have already been investigated by us ([3], [4]).

With non-minimal systems the following cases are possible.

a) Certain functions [$\varphi_k(x)$, for instance] do not belong to the space of the ot er functions of the system. Then in $L^2(a, b)$ there are functions

$$\varphi^k(x) = \frac{\varphi_k(x) - \hat{\varphi}_k(x)}{\varphi_{kk} - \hat{\varphi}_{kk}} \qquad (4')$$

satisfying the conditions $\int_a^b \varphi^k(x)\,\bar{\varphi}_i(x)\,dx = \delta_i^k \quad (i = 1, 2, \ldots)$

The number of these functions may be finite or even infinite. Thus, in this case to the system (1) there corresponds a degenerated biorthogonal system.

The functions $\varphi_k(x)$ above-mentioned cannot be discarded from the system (1) without decreasing its space.

In this case the system (1) is said to be partly minimal.

b) Every function of the system (1) belongs to the space of the remaining functions. No biorthogonal system corresponds to the system (1) and the latter is called uniformly non-minimal. Discarding any of the functions from this system will not decrease its space.

Furthermore there are such systems of functions in which every infinite subsequence has the same space with the function. They have been called quadratical dense systems([5]).

One may always make an attempt to minimize a non-minimal system. For this purpose those of the functions $\varphi_k(x)$ should be picked out from the system, for which there are functions defined by the formula (4'). The rest of the functions form a uniformly non-minimal system. Discarding any functions from it until it becomes a partly minimal system, we again select the functions that do not belong to the space of the remaining functions, and so on. The functions thus picked out form a minimal system, and if the process of minimalization is complete, the space of the minimalized system will prove to be the same as that of the system subjected to minimalization.

Received
11.I.1943.

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Comptes Rendus (Doklady) de Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

ASTRONOMY

PHYSICAL INTERPRETATION OF COLOUR PHENOMENA OF THE DISK OF JUPITER

By W. W. SCHARONOW

(Communicated by S. I. Wawilow, Member of the Academy, 18. II. 1943)

As is well known, the dark bands on the disk of Jupiter have a reddish hue, which is the most distinct near the centre of the disk; with the approach to the limb of the latter, it gradually disappears, passing into a neutral grey colour. The separate dark spots that appear on the disk of the planet display analogous phenomena and some of them possess a peculiar intense colour. The famous «red spot» which was observed at the end of the last century may serve as an example of this phenomenon.

Different hypotheses have been suggested to explain this red colour of the dark parts of the Jupiter disk. Formerly it was supposed to be due to radiation of the red-hot surface of the planet. Recently it has been assumed that the atmosphere of Jupiter contains some special organic compounds possessing a brown or red colour. It will be shown here that hypotheses of this kind are not indispensable and that the distribution of colour over the Jupiter disk can be explained by the usual scattering of light in uncoloured gases and aerosols.

Let us assume, as usual, that the atmosphere of Jupiter consists of a gas, scattering light according to the law of Rayleigh. At various heights of the atmosphere there are floating layers of a white opaque aerosol, which are similar to terrestrial clouds as to their optical properties. Products of condensation of methane or ammonia may well give formations of such a kind.

The apparent brightness B' of an object immersed into a turbid medium may be represented by the following empiric formula given by V. Faas([1]):

$$B' = M + Ne^{-x}. \qquad (1)$$

Here M is the value of the brightness of an infinitely thick layer of the atmosphere; $M+N$, the brightness of the object at the limit of the atmosphere, and x, a value proportional to the optical thickness of the layer. The multiplier e^{-x} is connected with the transmission factor of the gas layer lying above the object; it is not equal to this factor, however, and may be called the «effective transmission factor».

We shall denote the solar illumination at the distance of Jupiter by E, and the brightness of some element of the surface of the planet by B. The value

$$\rho = \frac{B}{E}$$

may be called the «reflectivity» of this element. If the apparent surface of the planet is a diffusely reflecting stratum lying under the layer of the gas, then, according to the equality (1), we may write:

$$\rho = \rho_a + (\rho_c - \rho_a)\, e^{-x} \qquad (2)$$

where it is denoted:

$$\rho_a = \frac{M}{E}\,;\quad \rho_c - \rho_a = \frac{N}{E}\,.$$

Here ρ_a is the reflectivity of an infinitely thick layer of gas, and ρ_c, the reflectivity of the clouds.

Let us assume that for the monochromatic radiation with wave-lengths λ_1 and λ_2 the reflectivity of the surface is expressed by the values ρ_1 and ρ_2. Then the colour of the surface may be expressed by the ratio:

$$C = \frac{\rho_1}{\rho_2}\,.$$

Let us put $\lambda_1 > \lambda_2$. Then $C > 1$ for a red object and $C < 1$ for blue ones; for a neutral or grey object $C = 1$.

If a single scattering of light takes place in the gas and in the aerosol, and there is no true absorption, or if there is absorption, but the ratio of the absorption factor to the extinction factor is constant over the spectrum, then the values ρ_a and ρ_c will be independent of λ with any law of change of the diffusion factor. In other words, an infinitely thick layer of such a turbid medium scattering the light according to the law of Rayleigh, or according to any other law, will reflect the light as would do a neutral grey surface. Therefore in the equality (2) only the value of the parameter x will be changed along with λ. This will give the following expression:

$$C = \frac{\rho_a + (\rho_c - \rho_a)\, e^{-x_1}}{\rho_a + (\rho_c - \rho_a)\, e^{-x_2}} = \frac{A + e^{-x_1}}{A + e^{-px_1}}\,. \qquad (3)$$

Here $A = \frac{\rho_a}{\rho_c - \rho_a}$ and the multiplier $p = \frac{x_2}{x_1}$ in the case of a purely gaseous atmosphere will be equal to $(\lambda_1/\lambda_2)^4$. It follows from equation (3) that with $x = 0$ and $x = \infty$ there will be $C = 1$, i. e. the reflexion proceeds neutrally. The same is true with any x if $\rho_c - \rho_a = 0$, i. e. if the reflectivity for the clouds and for an infinitely thick layer of the atmosphere is the same. In all other cases a grey cloud observed through the atmosphere will appear to be coloured. If its reflectivity is higher than that for an infinitely thick layer of the atmosphere, then $\rho_c - \rho_a > 0$, and with $p > 1$ (as it is in the case of diffusion according to the law of Rayleigh) we shall have $C > 1$, i. e. the apparent colour will be reddish. In the opposite case $\rho_c - \rho_a < 0$ and $C < 1$ and the colour of the object will be bluish. In both cases with the increase of x the colour will also increase at first, then with $e^{-x} = y$ it reaches its maximum, where y will be defined by the following equation:

$$y^p + \frac{Ap}{p-1}\, y^{p-1} - \frac{A}{p-1} = 0 \qquad (4)$$

after which it decreases again, asymptotically approaching the grey colour.

We may apply the above considerations to the explanation of colour differences on the Jupiter disk, if we suppose that the parameter ρ_a for the atmosphere of this planet is less than the reflectivity ρ_c of the clouds floating in the atmosphere. Under such conditions white clouds observed through the atmosphere will appear to be yellow or pink. This may serve to explain the yellowish colour of some other outer planets.

It is quite natural to suppose that in the dark places of the Jupiter disk the layer of the clouds lies deeper than in the light ones. Then their redder colour may be explained by a smaller value of e^{-x}. In order to explain the change of colour along the dark bands, it is necessary to assume that for the centre of the disk $e^{-x} < y$. Then with the approach to the limb of the disk the parameter e^{-x} is decreasing and the colour is approaching the neutral one.

We shall give here some calculations. The photometric observations of Jupiter, made at the opposition of 1938 (2), gave for the dark equatorial band, crossing the centre of the disk, the following values: in photovisual rays ($\lambda_1 = 555$ mμ) $\rho_1 = 0.590$,

in photographic rays (λ_2=420 mμ) ρ_2=0.440, hence C=1.34. For the above-mentioned combination of λ, and for the diffusion after λ^{-4} Rayleigh's law, we shall have p=3; after that we may represent the observations accepting the following constants for the equality (3):

$$A=1;\ \rho_c=0.84;\ \rho_a=0.42;\ e^{-x}=0.40.$$

The obtained value ρ_c is close to that given by the observations for the terrestrial clouds, but the low value ρ_a points to the presence of an appreciable absorption, since with scattering alone this value should be close to 1.

If it is possible to assume that with an angle of incidence of rays i the parameter x is changed proportionally sec i and the parameter A remains constant, then we shall obtain the following values for C at different distances from the centre of the disk (see the table).

i	C calculated	C observed
0°.0	1.33	1.34
5.8	1.33	1.33
11.5	1.32	1.32
17.5	1.31	1.32
23.6	1.30	1.26
30.0	1.27	1.25
36.9	1.25	1.24
44.5	1.21	1.19
53.1	1.15	1.14
64.2	1.12	1.06

The separate spots of an intensive red colour may be either clouds lying at some depth, for which e^{-x} is near to y, or else they may be parts of a cloud layer, lying at the same depth as the rest, but possessing a higher value of ρ_c. Thus, if we assume ρ_c=1 with values ρ_a, p and e^{-x} used above, then we shall obtain C=1.42.

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Received
15.II.1943.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

MATHEMATICAL PHYSICS

ON THE STABILITY OF INVERSE PROBLEMS

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Many objects of Nature have properties that are not well fitted to, or even altogether defy, direct investigation. In such cases some of their characteristics are studied whose manifestation can be measured. Our judgment on the structure of the earth crust, for instance, may in certain cases be based on the investigation of its characteristics, such as density, or electric conductivity, which determine the respective physical field accessible to measurement at the surface of the earth—the gravitational field, the field of electric current.

Given a certain characteristics of the medium (distribution of density or of electric conductivity), we are usually able to compute (precisely or approximately) the physical field determined by this structure in the region where the measurements are made. Yet, the problem to be solved here is the reverse to this. Namely, the physical field is known, while the structure of the medium which determines it is sought.

The customary way to solve the inverse problems is by selection. Within an arbitrarily chosen (sufficiently wide) class of possible structures of the medium the corresponding physical fields are computed and the solution of the problem arrived at by selecting some admissible medium, for which the calculated physical field shows but a small deviation from observation.

In order to put the method of selection upon a solid foundation, it is necessary to establish (or admit) the existence of certain regularities: 1) One has to establish the uniqueness theorem for the direct correspondence, i. e. to prove that no two different types of medium have a single corresponding field. Then we have also the right to speak of a reverse correspondence. Without this the method of selection has no sense at all. 2) The coincidence between the calculated and observed field is not an absolute one (if only because the selection is made in an approximate way). We therefore have, moreover, to prove the stability of the inverse problem (or the continuity of the inverse mapping), that is, to make sure that with slight deviation of the auxiliary field from observation the respective structure of the medium cannot, possibly deviate strongly from the actual.

In studying the stability of the inverse problem, a number of questions of qualitative and quantitative character may be raised.

In the present paper set-theoretical conceptions are applied to one of these problems, which consists in proving that under certain conditions the stability of the inverse problem is a direct consequence of the uniqueness theorem. We shall also apply it to the inverse problem of the potential and to the study of the continuous dependence of the solutions of ordinary differential equations upon the parameter.

1. In the theory of continuous mappings there takes place the following theorem [1, 2].

Let a set of elements $\{x\}$ forming a metric space R be mapped continuously upon another set of elements $\{x^*\}$ forming a metric space R^*. If the mapping $x^* = f(x)$

is one-to-one and continuous and if the space R is compact, then the inverse mapping $x=f^{-1}(x^*)$ is also continuous.

Definition. Let a set of elements $\{x\}$ be mapped by a function $f(x)$ on to another set of elements $\{x^*\}$: $x^*=f(x)$. This mapping is said to be one-to-one at the point x_0 if $x_0^*=f(x_0)\neq f(x)$ for any element x distinct from x_0.

It is easy to prove the following theorem.

Let a metric space R be mapped continuously on to another metric space R^*

$$x^*=f(x) \qquad (x\in R, \quad x^*\in R^*).$$

If this mapping is one-to-one at the point x_0 and the space R is compact, then the inverse mapping $x=f^{-1}(x^*)$ is likewise continuous at the point x_0^*.

The continuity of the inverse mapping is understood to mean that for any $\varepsilon>0$ there exists a $\delta(\varepsilon)$ such that if

$$\rho(x^*, x_0^*)<\delta(\varepsilon),$$

then

$$\rho(x, x_0)<\varepsilon$$

where x is any prototype of the point x^*.

Though we formulate these theorems for metric spaces, they also hold in a more general case.

2. Among the direct problems of the potential theory there is one that consists in computing for the surface $z=0$ the potential of a bounded body T filled up with a homogeneous mass of density μ and lying beneath that surface ($z<0$). We shall demonstrate that the inverse problem has a stable solution.

Suppose approximately that the position of the disturbing body T is known to be inside a given surface S. Let us examine the totality of bodies $\{T\}$ satisfying the following conditions:

1°. Each of the bodies T belongs to a given bounded surface S lying in the region $z<0$.

2°. Each of the bodies T is stellate with respect to its centre of gravity, so that the equation of the surface Σ, bounding the body T, may in the spherical system of coordinates with their centre at the point P be represented in the form

$$z=f(\varphi, \theta).$$

3°. Function $f(\varphi, \theta)$ has its derivative numbers bounded by a number M, common to all bodies of the class R.

Let us determine the degree of proximity of two different bodies T_1 and T_2 from the class R by means of the number

$$\rho(T_1, T_2)=\max\{\rho(P_1, P_2), \max|f_1(\varphi, \theta)-f_2(\varphi, \theta)|\}$$

where f_1 and f_2 are functions defining the surface equations of the bodies T_1 and T_2 with respect to their centres of gravity P_1 and P_2.

We shall prove the following theorem.

Whatever may be the degree of accuracy of ε and the class R of the bodies, such number $\delta(\varepsilon)$ may be indicated that if the values of the potentials (or of their derivatives) $V_1(x, y)$ and $V_2(x, y)$ for any two bodies T_1 and T_2 of the class R differ from each other at $z=0$ by less than $\delta(\varepsilon)$

$$|V_1(x, y)-V_2(x, y)|<\delta(\varepsilon),$$

then these bodies are separated by a distance less than ε

$$\rho(T_1, T_2)<\varepsilon.$$

In fact, with the notion of distance determined as $\rho(T_1, T_2)$, the totality of bodies of the considered class R form a metric space. If the distance between

the potentials (or their derivatives) $V_1(x, y)$ and $V_2(x, y)$ be evaluated by the number

$$\rho(V_1, V_2) = \max |V_1(x, y) - V_2(x, y)|$$

(the maximum exists indeed), then the set of functions $\{V\}$ presents a metric space R^*.

The correspondence between the bodies T and the values of their potentials (or derivatives) determines the mapping

$$V = f(T)$$

of the space R on to the space R^*.

This mapping is continuous and one-to-one, for, in virtue of a well known theorem by P. S. Novikov, no two different bodies T_1 and T_2, stellate with respect to their centre of gravity, may have equal potentials corresponding to them. And the centre of gravity has its position determined by the value of the potential at the surface (3).

The class under consideration, R, forms a compact family. Direct application of the theorem mentioned in § 1 just proves the theorem.

Modifying the notion of distance between the bodies or that of proximity of potentials (or of potential derivatives), one may easily establish other theorems of stability.

It is well to note that without the conditions of type 1° or type 3° the stability of the inverse problem does not hold any longer.

3. Carathéodory has established the following theorem on the continuous dependence of the solutions of a system of ordinary differential equations upon the parameter (4).

Let the functions $f_k(x; y_1, \ldots, y_n; t)$, $(k = 1, \ldots, n)$ be given, satisfying the conditions:

a) For every value of t inside a certain neighbourhood U_{t_0} of the point t_0 every function $f_k(x; y_1, \ldots, y_n; t)$ is measurable with respect to x when y_k are fixed, and is continuous with respect to $y_1, \ldots, y_n$ when x is fixed, x varying within the region $a < x < b$; and, moreover, there exists such a measurable (in the region $a < x < b$) function, independent of t, that

$$|f_k(x; y_1, \ldots, y_n; t)| < M(x) \quad (k = 1, \ldots, n).$$

b) For $t = t_0$ and arbitrary values of $y_1, \ldots, y_n$ the functions $f_k(x; y_1, \ldots, y_n; t)$ are continuous with respect to $y_1, \ldots, y_n; t$.

c) For $t = t_0$ and $\alpha_k(t_0) = \alpha_k^0$ there exists a sole system of functions $y_k(x, t_0) = y_k^0(x)$ satisfying the equations

$$y_k(x, t) = \alpha_k(t) + \int_x^x f_k(t; y_1(x, t), \ldots, y_n(x, t); t)\, dx \quad (k = 1, \ldots, n). \tag{1}$$

If, under these conditions,

$$\lim_{t \to t_0} \alpha_k(t) = \alpha_k^{(0)}, \tag{2}$$

then for any system of solutions (1) at any point of the region under consideration

$$\lim_{t \to t_0} y_k(x, t) = y_k^{(0)}(x).$$

Let us examine the totality of all possible systems of solutions of equation (1) when $t \subset U_{t_0}$. Defining the distance between the elements $Y_1 = \{y_1^{(1)}(x), \ldots, y_n^{(1)}(x)\}$, $Y_2 = \{y_1^{(2)}(x), \ldots, y_n^{(2)}(x)\}$ as

$$\rho(Y_1, Y_2) = \sqrt{[y_1^{(1)}(x) - y_1^{(2)}(x)]^2 + \ldots + [y_n^{(1)}(x) - y_n^{(2)}(x)]^2}$$

we obtain a metric space R which is compact by virtue of the obvious fact that the functions $y_k(x)$ are uniformly bounded and uniformly continuous in

their totality [this is a consequence of the existence of $M(x)$ and the condition (2) if only U_{t_0} is small enough].

To every element $Y = \{y_1(x), \ldots, y_n(x)\}$ of the space R we make correspond a point of an n-dimensional Euclidean space $\{\alpha_1, \ldots, \alpha_n\}$, putting

$$\alpha_k = y_k(x_0) \quad (k = 1, \ldots, n).$$

The distance between the images of two elements being less than that between the elements, it will be obvious that this is a continuous mapping.

In virtue of the conditions of the theorem this mapping is also one-to-one at the point $\{y_1(x, t_0), \ldots, y_n(x, t_0)\}$.

Applying the theorem of § 1, one sees that the inverse correspondence is continuous in the sense that for any ε there exists such $\delta(\varepsilon)$ that if

$$\sqrt{(\alpha_1^{(0)} - \alpha_1)^2 + \ldots + (\alpha_n^{(0)} - \alpha_n)^2} < \delta(\varepsilon),$$

then, whatever the solution $\{y_1(x), \ldots, y_n(x)\}$ corresponding to $\{\alpha_1, \ldots, \alpha_\varepsilon\} : y_k(x_0) = \alpha_k$ may be, we shall have

$$\sqrt{[y_1^{(0)}(x) - y_1(x)]^2 + \ldots + [y_n^{(0)}(x) - y_n(x)]^2} < \varepsilon$$

and this proves the theorem.

Many examples can easily be put on the stability of inverse problems.

Received
12.IV.1943.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

PHYSICS

ORIENTATION OF RECRYSTALLIZATION GRAINS IN ZINC CRYSTALS

By V. A. ZOLOTOV

(Communicated by A. F. Joffe, Member of the Academy, 12. IV. 1943)

1. From numerous experiments on recrystallization of deformed polycrystalline samples of various metals it is seen that there exists a definite relation between the initial texture of the sample ([1], [2]) and the preferential orientation of the grains grown in the course of calcining (the so-called «recrystallization texture»). On the other hand, experimental data are available, pointing either to a total lack of any regular relation between the texture before and after tempering, or to a highly intricate and contradictory nature of the relations ([1]).

Thus, the study of polycrystalline aggregates, carried on from this angle, gives no definite solution of the problem as to the conditions responsible for the orientation of new grains during recrystallization. This is because of the extreme complexity and diversity of the factors that may affect the orientation of new grains in a deformed polycrystalline material. As such may act: various heterogeneities of deformation, unavoidable curvatures and disturbances of the lattice (which are of particular importance near the intergranular boundaries), influence of admixtures, presence of the so-called «cumulative recrystallization», etc.

An important rôle in the interpretation of recrystallization textures must therefore belong to experiments with recrystallization, and to a subsequent study of the orientation of crystallization grains in metallic monocrystals. The nature of the relation between orientation previous to and following tempering may be studied with the greatest ease in such crystals which show no large number of initial orientations. For instance, a monocrystal of Zn deformed under definite conditions shows no more than two main orientations: that of a normal crystal and of twinned regions of the lattice ([1]). A study of the orientation of new grains in such a sample would make it possible to bring out the effect of either of them, in particular that of the orientation of a crystal in deformation twins, which play an important part in the recrystallization of zink ([3]).

Such experiments would also be helpful in the solution of another, no less interesting and controversial problem: that of distribution of internal potential energy in a twin crystal. A number of experimental data point to the fact that the twin interlayer is more saturated with energy than the neighbouring areas of a normal crystal. Such data are: the preferential growth of new grains during recrystallization inside of twins ([4]), the high solubility of twin formations ([5]), experiments with diffusion of Hg in Zn ([6]), etc. The authors of the papers cited above have been brought to the conclusion that deformation twins as a whole are systems with an internal energy higher than that of other regions of a deformed crystal; in all cases they represent a strained unstable phase. There is, however, quite a number of arguments in favour of the opposite point of view. Indeed, mechanical twinning of a crystal can hardly be imagined otherwise than as a phenomenon of a leapwise

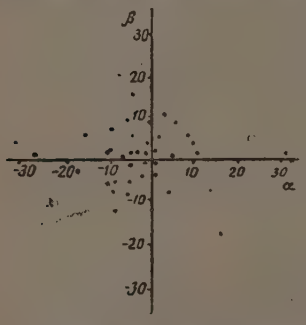
discharge of stresses, which is made quite illustrative by extension diagrams of Zn and Cd crystals, pointing to a sharp drop in the load after the formation of each twin. A twin may therefore be thought of as an equilibrial stable phase with a smaller content of energy than the neighbouring strained areas of the crystal, where no discharge of stress has taken place still. Not unfrequent are instances of twinning at a natural growth of crystals or in the case of recrystallization, viz. in the course of typical processes leading to the establishment of one of the forms of equilibrium state of the crystalline lattice. The fact that the deformation of twinning requires a certain expenditure of work only up to a given moment, after which the process goes on spontaneously ([7]), also gives ground to suppose that in the course of twinning the crystal lattice passes from one stable state to another. The twinning theories available are also based on an analogous assumption ([8]).

The solution of the problem as to the distribution of internal potential energy in a twinned crystal of Zn can be associated with the study of the orientation of grains formed during the tempering of such a crystal, since the presence of a given preferential orientation will indicate the foci of the most intense recrystallization. And as soon as the zones of preferential formation of new grains will have been established, there will be ground to estimate their energy content, assuming that the centres of recrystallization correspond to the points of the maximum gradients of the thermodynamical potential of the crystalline lattice.

2. In the experiments use was made of zinc crystals obtained after the method of Bridgeman at the physical laboratory of this Institute. For the manufacturing of the samples there were chosen crystals with a large angle (approaching 80°) between the edge (0001) and the axis. From such crystals cylindrical plates were split off: the plates were made 2—3 mm thick and had as a base the mirror-surfaces of splitting along the edge (0001) of the base of a hexagonal prism, on which recrystallization was observed in the subsequent. The choice of such plates as an object of direct observation made it possible to obtain on a considerable number of samples irreproachably pure base planes, almost free from any distortion. This was of great importance for the statistical treatment of the experimental data, permitting to vary on a large scale and to choose optimal conditions for deformation and tempering. The problem of adjusting the crystals for goniometrical examination was also facilitated. As a result of a slight bending of the plates there was formation of twins (no more than 2—3 in each), the traces of which could easily be seen on both bases as rather narrow bands. The plates thus deformed were tempered at 350° C. Upon 5 minutes of heating the samples were every time removed from the beaker, etched with strong HCl and examined under the microscope. Such plates as showed the first signs of recrystallization were sorted out; the rest were tempered at the same temperature during 5 minutes more, etc. By means of this «stepwise» heating we succeeded in tracing the initial stages of the growth of grains. A sufficient number of samples was obtained, on which it was possible to see distinctly under the microscope both the traces of twins and the new grains overgrowing them. Their respective orientation could therefore easily be established. The orientation was determined by the method of «maximum brilliancy» with the aid of a microscope supplied with a special rotating stage. The sample was fixed to the stage by means of a paste. The stage was adjusted in such a way that a maximum of reflected light be obtained at first from the surface of the twin, illuminated by a bright beam of parallel rays, and next from the surface of the grain grown out of the twin. The angle between the normals and the edges (0001) made apparent by etching was determined from the angles α and β of the stage rotation in two mutually perpendicular planes. As shown by experiment, in zinc these edges are those endowed with maximum reflecting capacity. For the purpose of the given problem it was quite sufficient to determine the angle between the directions of the hexagonal axes of the lattice in the twin and in the grain grown from the latter in order to form a judgment on the preferential orientation of the new grains. If the orientation of the recrystallization grains was a random one, bearing no relation to the original orientations, this would be manifest in disorderly orientation of the directions of their hexagonal axes. Without being very accurate, the method used for determi-

ning the orientations excels the other methods in that it makes it possible to determine the orientation of microscopically small (0.02 mm across) grains. This cannot possibly be achieved by any other method. We determined the orientation of all recrystallization grains except those which directly adjoined the edges of the sample, or fissures or some casual heterogeneities, or such as showed some faults of etching upon their surface and formed compact groupings; the orientation of some of these grains could have been affected not so much by the initial orientation of the crystal, as by secondary phenomena, arising at the edges of the grains in the process of their growth.

A total of 43 measurements were carried out. The results are represented in the figure, where along the axes of coordinates are plotted angles α and β of the deviations of directions of hexagonal axes of the lattices of new crystallites from their original position, which corresponded to the direction of the axis of the twin lattice. As seen from the figure, the directions of the axes in the new grains are tightly grouped around the original position 0; they mostly deviate from the twin axis by no more than 10°, including the error of measurement. Cases of widely differing orientations are comparatively rare. It can therefore be stated that the new grains, formed as a result of zinc recrystallization, show a certain preferential orientation, coinciding with that of the twinned areas of the lattice from which the grains have grown. Although analogous experiments, involving the study of the orientation of grains upon a cylindrical surface of zinc crystals, yielded a somewhat more irregular distribution of orientations, this run of experiments also showed a sufficiently distinct preferential orientation.



3. The fact that the crystals of zinc have been found to show a preferential orientation of the recrystallization grains, which coincides in the main with the orientation of the lattice in deformation twins, justifies the following conclusion: in the case of recrystallization, i. e. when a grain is being formed inside of a crystalline phase already present, the orientation of the new grain is no random one; it is related in a definite way to the orientation of the lattice in that region of the crystal, where the new grain has arisen.

Being on the whole one of the forms of the equilibrium state of the crystalline lattice, the twin necessarily shows in its portions, adjoining the body of the original crystal, some distortions of the crystalline lattice. These distortions consist, in particular, in bends of the layers of the crystal adjacent to the boundary. The existence of such distortions is evidenced by a number of facts. In zinc the foci of recrystallization are always concentrated near the boundaries of the twin interfaces ([9]). When dissolving the surface of a zinc monocrystal in a weak solution of HCl, we observed with the aid of a binocular magnifying glass deep etching furrows along the very boundaries of the twin bands. Twinning is accompanied with the appearance of diffuse spots ([10]) on the Laue-spectrograms. Numerous investigations concerning twinning of ionic crystals also point to disturbances in the crystal structure near the twin boundary ([7],[11]), etc. In the distorted boundary regions of the twin there take place simultaneously other bends of the crystal layers; separate small regions of the crystal (blocks) show plastic turns with respect to one another. The excess of potential energy is accumulated in the form of elastic stresses in those bent layers. The blocks present very small, isolated and somewhat desoriented fragments of a crystal, formed in the process of deformation; in most of the cases they preserve a regular internal structure. The potential energy of these portions of the crystal is minimum; being in direct contact with the neighbouring regions that have suffered elastic bending, when heated, these parts of the crystal start growing at the expense of their unstable deformed surroundings. Acting as a peculiar «crystalline nucleus», they play the part of primary cells of recrystallization. This accounts for the fact that new grains mostly assume the orientation of

the twins. A certain irregularity in the orientation (see figure) is due to the fact that the primary crystalline cells (blocks) are somewhat desoriented.

The following experiments were carried out in order to make clear the part played by elastic bends in the recrystallization of zinc: 10 zinc monocrystals were subjected to bending; the crystals became uniformly covered with twin traces throughout their length. After this each crystal was cut into two parts, one of which was tempered during a long period of time at 150°C. The other half of the same sample was not subjected to this treatment. Five minutes of tempering at 350° proved sufficient for the second halves of all the crystals to display all over their surface the usual picture of recrystallization. Unlike this, the first halves, that had been subjected to preliminary «rest», altogether failed to recrystallize under similar conditions of tempering, or else showed an obviously suppressed recrystallization capacity. This can be accounted for by a reduction in the differences between the potential levels at the boundaries between the blocks and the adjoining portions of the crystals, that have suffered elastic deformation, due to some relaxation of the elastic stresses during rest.

The internal perfection of twin's lattice is nevertheless somewhat impaired in the process of twinning. This is made evident by experiments involving manifold twinning of calcite ([7]). It is therefore quite possible that in the case of recrystallization new grains may also grow into the twins ([4]); if the orientations of the grain and the twin coincide more or less, the conditions created are especially favourable for the formation of new edges. In this case there takes place a certain thickening of the twin interlayer, which has been pointed out by a number of authors ([3], [12]).

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Received
19.XII.1942.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

PHYSICS

ON THE ORIGIN OF COSMIC RAYS AND ON THE MEAN LIFE-TIME OF ATOMS

By B. FESSENKOFF, Member of the Academy, and N. PARIJSKIJ

According to the recent work of Millican, Neher and Pickering (1), the primary cosmic rays are characterized by an enormous energy, corresponding to different levels numerically obtained as an energy of atomic annihilation.

As computed by Epstein, the magnetic field of the Earth produces a partly blanketing effect so that the rays originated by annihilation of helium are capable of penetrating into the Earth atmosphere only at +56.°7 of magnetic latitude, the rays corresponding to carbon, at +42°, etc.

It may be supposed that the atoms are transformed spontaneously into pairs of negative and positive electrons; and that this process is, perhaps, independent of the physical property of the surrounding medium. Nevertheless, the very hard emission produced by this process in the interior of the stars is transformed after a long chain of interactions into a very soft emission superposed upon their ordinary visible radiation. If, on the other hand, annihilation takes part in a practically empty space, the primary energy of the cosmic rays is fully preserved and may occasionally be observed as such. In an intermediate case, when the interstellar space is not totally transparent, the cosmic rays may partly contain the photons insensitive to the magnetic fields of the Earth and softer electronic components. The total quantity of the cosmic energy observed at the Earth surface is 3×10^{-3} ergs per cm^2 sec., that is nearly one half of the total energy from the stars of our Galaxy(2).

It may be proved that the cosmic rays have their origin in the Metagalaxy. Let us compare the intensity of cosmic radiation J, as it is observed at the Earth's surface, with that portion J_1 of the ordinary Sun radiation which may be attributed to the same process of annihilation. We have

$$A = \frac{J}{J_1} = 3 \frac{l^2 \bar{N} p}{r^2 N_1 p_1}$$

where r, l are the radii of the Galaxy and the Earth orbit; N, N_1, the total number of atoms in the intergalactic space and in the Sun; and p, p_1, the corresponding probabilities of annihilations. It may be supposed that $p = p_1$. The mass of intergalactic space involved is about 2×10^{10} $\odot$. Then it appears that the rate of annihilation is approximately 2 ergs/g sec. This is utterly impossible. The mass of diffused matter which produces the hard cosmic radiation must be many times greater and, consequently, exceed greatly the limits of our Galaxy.

Another evidence of the metagalactic origin of the cosmic rays is their uniformity all over the sky. Indeed, the Milky Way adds nothing to their intensity when the counter is directed toward the centre of the Galaxy. This remarkable phenomenon may be explained in different ways. It can be shown that our Galaxy is nearly fully transparent to cosmic radiation; secondly, that the Metagalaxy is very great

and its optical mass in every direction is not small. On the other hand, the same conclusion may be arrived at if the matter in the metagalactic space is in monochromatic radiative equilibrium with cosmic radiation.

Let us consider according to Millican and Wise the abundance of different elements in space (see the table).

No	Elements	N_i, %	A	Z	$\overline{A}$	$\overline{Z}^2$	$\overline{N}_i$, %	Abundance in % of mass and energy	Absorption in %
1	H	88.00	1	1	1	1	88.00	48.2	19.5
2	He	8.80	4	2	4	4	8.80	19.2	7.8
3	C	0.88	12	6	14	47	2.46	18.8	25.6
	O	0.88	16	8					
	N	0.44	14	7					
	B	0.26	11	5					
4	Si	0.26	28	14	27	190	0.49	7.4	20.6
	S	0.09	32	16					
	Ne	0.09	20	10					
	P	0.03	31	15					
	Al	0.02	27	13					
5	Fe	0.02	55	26	47	480	0.25	6.4	26.5
	V	0.03	51	23					
	Cl	0.02	35	17					
	A	0.01	40	18					
					1.83	4.5	100.0	100.0	100.0

We may arrange them in a limited number of natural groups according to their atomic weights. Although the number of hydrogen atoms is 88%, their mass is only 48.2%. The silicium band whose energy, as measured in India, is 0.23×10^{-3} ergs/cm^2 sec., furnishes some 7.4% of the whole mass. The rôle of the heavy atoms in absorption is much greater (see last column of the table), being proportional to the square of charge. The absorption of cosmic rays may be calculated according to the ordinary formula

$$I = I_0 e^{-\tau_0 x},$$

because the energy of annihilation is many thousand times greater than the value mc^2 corresponding to the electron. The optical mass is then

$$\tau_0 = N_0 \Phi = N \overline{\Phi} \left[4 \ln \frac{183}{\sqrt[3]{Z}} + \frac{2}{9} \right], \quad \overline{\Phi} = \frac{\rho_0^2 \overline{Z}^2}{137},$$

where N_0 is the number of atoms in unity of volume; ρ_0, the classical radius of the electron (2.80×10^{-13} cm); Z^2, the mean value of the square of the atomic charge.

The ratio $\Phi/\overline{\Phi}$ is nearly independent of the chemical composition of the medium (for hydrogen 20.96, for silicium 17.44). We put it equal to 18.3. Consequently

$$\tau_0 = 18 \frac{\rho_0^2}{137} N_0 \overline{Z}^2.$$

Taking $\overline{Z}^2 = 4.5$; $\rho_0^2 = 7.84 \times 10^{-26}$, we obtain

$$\tau_0 = 4.62 \times 10^{-26} N_0.$$

As the upper limit of intergalactic density one may take 10^{-24} g/cm^3. Then the maximum value of τ is 4.43×10^{-5} per kiloparsec. The Galaxy is then nearly transparent to cosmic radiation.

But it may be pointed out that in the Galaxy some accidental accumulation of matter may be present. Nevertheless, its effect of absorption is nearly exactly balanced by the effect of emission, if the optical mass of Metagalaxy is not small. Indeed, let ε, τ_0 be the coefficients of emission and absorption. The quantity of energy observed is then

$$I = \int_0^{\tau} \frac{\varepsilon}{4\pi} e^{-\int_0^x \tau_0 dx} dx.$$

It may be supposed that

$$\frac{\varepsilon}{4\pi} = \beta\tau_0, \quad \beta \text{ being constant.}$$

Consequently

$$I = \beta \int_0^{\tau} \tau_0 e^{-\int_0^x \tau_0 dx} dx = \beta(1 - e^{-\tau}).$$

This expression is independent of the mass distribution along different directions when $\tau \gg 1$. Accordingly, the cosmical rays originate in the great Metagalaxy. Their intensity per 1 cm^2/sec. is given by the expression

$$I = \frac{1}{4\pi} \cdot \frac{\varepsilon}{\tau_0} = \frac{\gamma p N_0}{4\pi\tau_0}$$

where $\gamma = mc^2$, the energy of annihilation of separate atom; N_0, the number of atoms; p, the probability of annihilation. The coefficient of absorption per one atom, representing the mean constituent of the medium, is

$$\frac{\tau_0}{N} = \Phi.$$

The mean life-time is then

$$t_0 = \frac{1}{p} = \frac{mc^2}{4\pi\Phi I} = 1.6 \times 10^{17} \text{ years}$$

$$\left(c = 3 \times 10^{10} \text{ cm}; \quad I = \frac{3 \cdot 10^{-3}}{\pi} \text{ ergs/cm}^2 \text{ sec.}\right).$$

The same methods may be applied to the evaluation of value of t_0 for separate emissions. As for the silicium group $I = 0.23 \times 10^{-3}$ ergs/cm^2 sec., we obtain

$$t_0 = 1.5 \times 10^{17} \text{ years},$$

i. e. nearly the same value.

The recognition of the spontaneous annihilation of matter leads directly to many consequences of astronomical importance.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

HEAT ENGINEERING

METHODS DEVELOPED FOR CALCULATING THE CIRCULATION OF WATER IN STEAM BOILERS

By M. A. STYRIKOVICH

(Communicated by M. V. Kirpichev, Member of the Academy, 15. I. 1943)

The first complete method for calculating the circulation in water-tube boilers is known to have been evolved by Münzinger (1). His computations were based on the motion of a steam-water mixture conceived as a homogeneous liquid, so that relative velocity was supposed to be absent. In that way the weight of a column of steam-water mixture and, accordingly, the useful head of the circulation came to be determined by calculation.

Later on it was shown by Schmidt and collaborators that the relative velocity of the steam strongly increased the head density in comparison with the discharge density. Schmidt, however, was able to produce useful data for design purposes in the case of immovable water only. For practical conditions, that is, when the water is moving, the data by Schmidt and Schurig were given in the form of the dependence of relative velocity upon the density of the mixture and the degree of throttling of the flow (expressed in terms of the throttle valve opening).

It may easily be understood that this value, being an individual characteristic of the experimental circuit, was not fit to be used in the calculation of water circulation in steam boilers. Moreover, Schmidt and Schurig carried out their experiments mainly with a tube of one diameter when the discharge of steam was very low and pressure did not exceed 40 atm.

The question as to the methods employed in boiler construction remained open until 1940, when Van Brunt, vice-president of the Combustion Engineering Corporation, reported to a meeting of the ASME the method adopted by that firm for the calculation of water circulation in water-tube boilers (2).

This method turned out to be merely a modification of Münzinger's and likewise failed to make allowance for relative velocity and the slipping losses connected with it, although it had been known since the experiments by Schurig that these losses must, at least under certain regimes, be very substantial. In this connexion a somewhat increased value seems to be taken for the coefficient of friction, which partly compensates for the failure to take slipping losses into account.

Thus, the methods still used abroad are based on ordinary hydraulics, whereby no allowance is made for the specific conditions peculiar to the movement of a two-component flow.

A different line has been followed by those engaged on the same problem in this country.

As early as within 1932—1934 the author together with several other workers (Shapiro, Baldina, Makarov and, especially, Peterson) started systematic experiments at the Central Boiler and Turbine Institute (CBTI), in which a series of special stands was used for the study of the circulation (3).

These experiments have brought into evidence the existence of a unique relation between the visible head density of the mixture and the reduced velocities of the steam and water, which are obtained by referring the discharge of the respective component to the entire cross-section of the tube. Pressure within 3 to 40 atm. was found very slightly to influence the visible head density (i. e. head density plus resistance to the movement of the mixture flow).

The experiments have covered a rather wide range of reduced velocities of water and steam. There was no need therefore to have recourse to considerable extrapolation, while computing the circulation for ordinary boilers with tubes varying in diameter from 55 to 80 mm, and working under a pressure up to 40 atm.

This method of calculation has been elaborated by D. F. Peterson and the author and adopted since 1936 by our boiler constructing industry.

The necessity to design boilers with tubes of a small diameter (for marine engines) and others working at a high pressure urged the organization of new experiments.

A long series of tests (not published as yet) with small-diameter tubes (23 to 32 mm) was carried out by the CBTI in 1936—1937. In these tests, beside vertical tubes, those inclined at different angles to the horizon have been used in determining the visible head density, when steam-water mixture is moving along the tubes.

In 1939—1940 other experiments were arranged at that Institute covering a pressure range of 10 to 180 atm. with tubes 25.5 and 56 mm in diameter(4). Nearly at the same time workers of the All-Union Thermotechnical Institute (AUTI) were conducting experiments with pressures ranging from 50 to 140 atm. and tubes 30 mm in diameter (5).

A large accumulation of new experimental evidence justified an attempt to connect together all data furnished by the above-mentioned investigations in order to lay a foundation for such designs of the circulation as would suit different types of boiler (stationary small-tube and marine boilers) working at low or high pressure.

All along it was necesary to establish how far it was possible to use the methods of calculation built upon ordinary equations derived for a homogeneous liquid, since that way, aside from a certain simplification of computations, might open the possibility for passing by means of some formulae to conditions not embraced by the experiment. This attempt has led to results that are partly exposed below.

The experimental data were compared to each other and to the values of the useful head of circulation computed without allowance for slipping. Their convergence was satisfactory. Only the experiments made in CBTI with a 23-mm tube (1936—1937) gave much lower values of the useful head at circulation velocities above 1.0 m/sec., as compared to the data obtained in the same tests with a 32-mm tube and to those obtained in 1939—1940 with a tube measuring 25.5 mm in diameter. Apparently, the data for the 23-mm tube should be discarded for $W_0 > 1$ m/sec., though it is, of course, desirable to repeat the investigation of this region.

A good quantitative agreement was stated also between the data obtained in these tests with a steam-water mixture under moderate pressure and those on air-lifts, when the viscosity of the lifted liquid was not very high. Still more remarkable was the coincidence of the experiments made with a steam-water mixture and with a mixture of mercury vapour and mercury(6).

The relation between the useful head of circulation and the different parameters of the process may briefly be characterized as follows:

1) The variation of the useful head with the diameter is very slight (Fig. 1), all the more so when the velocities of steam and water are high. Analytical calculations, according to Münzinger and Van Brunt, give a considerably stronger dependence.

2) Also the rate of circulation has, as a rule, less influence than would be consistent with the figures yielded by the analytical methods (Fig. 2), and only for very high reduced velocities of the steam, $W_0'' = 7—8$ m/sec., does the experimental dependence come near the computed one.

3) The influence of pressure is much below that obtained on the basis of analytical methods (Fig. 3).

With W_0 and W_0^r given the useful head is nearly independent of pressure in spite of a sharp decrease in water density. The result is that the rate of circulation upon constant heating does not decrease, but rather grows with pressure up to 180 atm. This is confirmed both by the experiments made in the Central Boiler and Turbine Institute (within 10 to 180 atm.) and by those carried out in the All-Union Thermotechnical Institute (50 to 140 atm.).

Thus, the loss value (friction plus slipping) proves very low in the region of high pressure. What appears mainly to

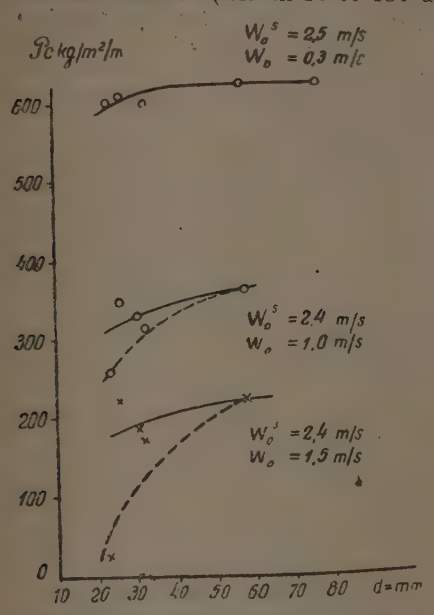

Fig. 1. Variation of the useful head of circulation with the diameter of the tube. Pressure 10—12 atm. Continuous lines, experimental data. Broken lines, data calculated according to Van Brunt (the values corresponding to the largest diameter are arbitrarily brought in coincidence)

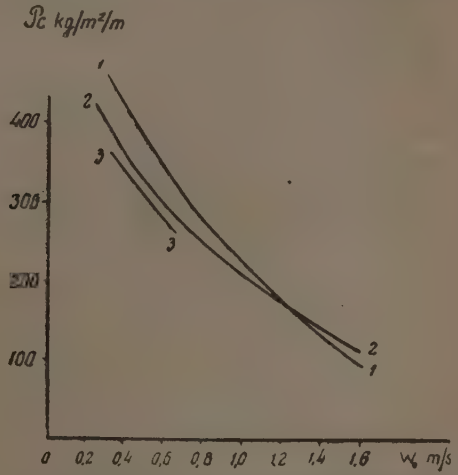

Fig. 2. Useful head as a function of circulation velocity at high pressure (110 atm.) and low reduced velocity of the steam ($W_0^r = 0.8$ m/sec.). *1*—according to Van Brunt; *2*—experimental data of the CBTI, 1940; *3*—experimental data of the AUTI, 1940—1941

account for this is that the relative velocity of the steam goes down as pressure grows. With low rate of circulation, when the losses by friction are slight and, accordingly, the value of relative velocity can be computed directly from the experimental data for the visible head density of the steam-water mixture, the decrease with pressure has been observed in all tests.

It is of interest to point out that, according to this author's computation, the decrease of W_r by far exceeds the change which may occur owing to a reduction of the density difference between steam and water. Apparently, the main reason for the reduction of W_r is a higher dispersion of the steam in the mixture consequent upon the decrease of surface tension.

In the region of low pressure a decrease of bubble size at the point of detaching with increasing pressure was observed by Zysina of the Central Boiler and Turbine Institute.

The gentle slope of the $P=f(p)$ curves for W_0=const and W_0^r=const may, besides the above-mentioned physical factors, be partly explained by a certain arbitrariness of the quantity W_0 itself.

On account of evaporation the actual discharge of water across a given cross-section of the tube is always somewhat below that determined from the value of W_0 (i. e. from inlet velocity). This difference, though negligibly small at low pressures, is apt in the case of high pressure to decrease the density of the mixture quite appreciably. Thus, with $W_0 = W_0^r$ the friction due to dryness goes down from $\sim$0.5 to 0.43, as pressure increases from a low level up to 180 atm., respectively.

To sum up, it may be anticipated that a transition from 100 atm., the pressure level adopted in USA, to extra-high pressures of about 180 atm. should give a much lower reduction of useful heads than that determined from analytical calculations, and the velocity of circulation under ordinary conditions, far from decreasing, will even somewhat increase.

It is interesting to note that, according to preliminary communications ([7]), the first extra-high pressure boiler with natural circulation (180 atm., Twin-Branch station, USA) was found to have a circulation velocity greatly exceeding the calculated value and even higher than in many of the boilers working at moderate pressures.

So it must be stated that the methods of calculating the circulation now adopted outside this contry (particularly in USA) as «the most logical methods» can be used in the region unexplored by the practice only at a grave risk. To offset the errors by correcting the computation factors is a matter of considerable difficulty, because the corrections depend upon nearly all the parameters of the circulation.

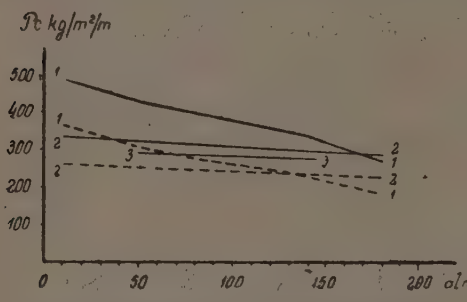

Fig. 3. Useful head as a function of pressure. Diameter of tubes, 25.5 mm. Continuous lines, circulation velocity, 0.5 m/sec.; reduced velocity of steam, 0.7 m/sec. Broken lines, circulation velocity 1.0 m sec.; reduced velocity of steam, 1.0 m/sec. *1*—according to Van Brunt; *2*—experimental data of the CBTI, 1940; *3*—experimental data of the AUTI, 1940—1941

Therefore in his present attempts to develop a better method the author has thought it expedient to give up the use of any analytical computations, and build the whole design on experimental values for the useful head of circulation. He has found, moreover, that the experience gained in other domains of technical sciences, especially the abundant experimental material on air-lifts, may be brought to bear on the subject. This at once affords the possibility of refining the design of air-lifts and creating an approximate method for the calculation of evaporators other than those of water, for which no satisfactory method of calculating the circulation has been devised at all.

Aside from the direct use of the available data, the generalization of the material obtained under different conditions permits to gain a deeper insight into the physical laws underlying the motion of two component flows, whereby the approach to working out their hydrodynamics will be made possible.

Not until this vast problem is solved, at least in first approximation, can an attempt be made to evolve the analytical design of the circulation in so comprehensive a form that it would take into account the peculiar physical conditions, under which the movement of two component flows is known to proceed. While the best course to be taken for the time being is to place the practical computation on a purely empirical basis.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

PHYSICAL CHEMISTRY

ON THE THEORY OF THE AUTOGENESIS OF CHAIN REACTIONS

By N. S. AKULOV, Member of the Academy of Sciences of the White Russian SSR

The length of the chain (ν) is the main conception of the modern theory of chain reactions, developed in the works of Bodenstein, Christiansen and Cramers, Semenov and Hinshelwood.

The most simple and strict way to find an expression for ν is the following. Let δ be the probability of the active centre entering the reaction; and $\varkappa$, the number of new active centres that will arise in this case. Then after the first link one centre will give rise to $\varkappa\delta$ centres; after the second link, $(\varkappa\delta)^2$; after the third one, $(\varkappa\delta)^3$, etc. Summing up and supposing $\varkappa$ and δ to be independent of the stage of the reaction, in case its course was isothermal, $\varkappa = \text{const}$, $\delta = \text{const}$, we shall find

$$\nu = \frac{1}{1 - \varkappa\delta}, \quad \text{where } \varkappa\delta < 1. \tag{1}$$

Within the so-called area of inflammation and explosions, when $\varkappa\delta > 1$, equation (1) has no sense. The area, however, includes not only all the reactions of combustion, inflammation and explosions, but, as we shall see, the totality of slow autogenetic reactions as well. Hence the necessity of introducing a new equation for the length of the chain, applicable in the case $\varkappa\delta > 1$.

The length of the chain is determined by the number of the molecules of the final product, and the number of chains at every given moment by the number of existing active centres. Therefore, if the number of the latter is constant, we must express the length of the chain as the ratio of the concentrations of the final products α (the total length of all the chains at the moment given t) to the number of chains, determined by the concentration of the intermediate product ε (as unit of concentration we accept the concentration of the final product at the end of the reaction). This simple expression for ν is possible only with $\varepsilon = \text{const}$. In the general case it is necessary to introduce into this examination the average

$$\bar{\varepsilon} = \frac{1}{t} \int_0^t \varepsilon \, dt \tag{2}$$

and to express the length of the chain by means of the equation

$$\nu^* = \frac{\alpha}{\bar{\varepsilon}}, \tag{3}$$

where α and $\bar{\varepsilon}$ are related with the moment t. The value ν^*, which in order to distinguish it from the stationary we shall call resultant, is determined directly from experimental data (which is one of its advantages). Nevertheless, for the analysis of the mechanism of reactions, i. e. for reducing the values observed experimentally to parameters that characterize elementary acts of the reaction, ν^* is to be expres-

sed as a function of $\varkappa$ and δ. With this purpose in view we shall write the equation of the balance of active centres

$$\frac{d\varepsilon}{dt} = \varepsilon_i' + (\varkappa\delta - 1)\frac{d\alpha}{dt} \tag{4}$$

where ε_i' is the total velocity of the spontaneous origination and break-down of active centres; $\varkappa\delta - 1$ is the mathematic expectation of that a centre which had entered the reaction and perished, will give rise to a new centre*.

Integrating (4), we obtain

$$\varepsilon = t\overline{\varepsilon_i'} + (\varkappa\delta - 1)\,\alpha. \tag{5}$$

However, we arrive at an equation of the same type if the expression for the length of the chain (3) is used. In fact, differentiating the integral by its upper limit, we have:

$$\varepsilon = t\,\frac{d}{dt}\left(\frac{\alpha}{\nu^*}\right) + \frac{1}{\nu^*}\,\alpha. \tag{6}$$

The existence of (5) in connexion with (6) imposes upon ν^* certain conditions, viz. owing to the identity of the left parts the right ones must also be identical. In particular, this is possible in the case the coefficients with t and α are equal and independent of α and t. This condition leads actually to a complete agreement of the conception introduced for ν^* with the equation of balance (4). But since a priori it is not the only possible one, we shall assume that it is just this simple condition that corresponds to the mechanism of chain reactions, introducing it as a postulate (unique and general for all the dynamics of chain reactions). In this way we have

$$\nu^* = \frac{1}{\varkappa\delta - 1}, \tag{7}$$

where $\varkappa$ and δ (and therefore ν^* as well) are magnitudes, independent of the stage of the reaction (in case of its isothermal development).

Equations (3) and (7), the first of which is a definition and the second is proved as a possible one and postulated as the only possible one, are the main equations for the new theory of chain reactions, establishing their mechanism. As will be shown below, they enable to deduce all the fundamental laws of the new chemical dynamics ([3]), as well as to build a new general theory of combustion, inflammation and explosion, free from the difficulties of the former conception, and agreeing quantitatively with the experiment.

First of all, by substituting (6), in the case $\nu^* = \text{const}$, into the law of mass action**

$$\frac{d\alpha}{dt} = \frac{1}{\Delta\tau_0}\,\varepsilon\,(1-\alpha) \tag{8}$$

where $\Delta\tau_0$ is the time of the development of the link in the initial stage of the reaction ($\alpha \to 0$), we find that the rate of the reaction is determined by the superposition of two processes, the autogenesis and the autocatalysis, the rates of which

$$\left(\frac{d\alpha}{dt}\right)_g : \left(\frac{d\alpha}{dt}\right)_k = \frac{d\alpha}{dt}\,t : \alpha \tag{9}$$

[main thesis of the new chemical dynamics ([3])]. The first of these two processes corresponds to the birth, and the second to the development of the chains.

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* The validity of this equation is evident. It can be obtained like (1) from the equations of Christiansen and Cramers, as well.

** As usual, we omit here the term $\frac{1}{\Delta\tau_0}\,\varepsilon\,(\varepsilon_0 - \varepsilon)$, where $\varepsilon_0 = \varepsilon_{\alpha=1}$, owing to its smallness. Nothing is changed if it is taken into consideration in equation (9). In a general case (if ε is not too small) its evaluation can be achieved by the method of successive approximations.

Further, dividing (8) by $1-\alpha$ and integrating, we obtain:

$$-\ln(1-\alpha)=\frac{1}{\Delta\tau_0}\bar{s}t. \tag{10}$$

Hence by means of (3) we find the law of autogenesis, previously stated by us on the basis of independent considerations:

$$-\ln(1-\alpha)=\alpha\frac{t}{\tau} \tag{11}$$

where

$$\tau=\nu^*\Delta\tau_0 \tag{12}$$

is the period of induction (time of delay of the jump in the rate of the reaction). Hence, taking into consideration (7) we find a new very important equation, which will serve us as a basis for the whole theory of combustion, inflammation and explosions:

$$\tau=\frac{\Delta\tau_0}{x\delta-1}. \tag{13}$$

The law of autogenesis (11) has been obtained for the case when $(\varepsilon)_{t=0}=0$. However, if in the system there is a catalyst that acts as a corresponding intermediate product with a constant concentration σ_0, we obtain from (10) after integrating

$$-\ln(1-\alpha)=(c_0+\alpha)\frac{t}{\tau} \tag{14}$$

where

$$c_0=\sigma_0\tau. \tag{15}$$

Thus, upon due generalizations and having introduced more precision into the main conception of the chain theory, an analysis of the mechanism of chain reactions brings us to the deduction of the general law of catalytically disturbed autogenesis (catagenesis).

Taking again equation (7) as the initial one, we obtain $\nu^*=\text{const}$. We come to the same equation directly on the basis of (3) and from the analysis of experiments of Bone and Hill for the kinetics of the oxidation of ethane. Further, the treatment of 40 experimental kinetical curves on the basis of (14) showed them to be in quantitative agreement with theoretical curves at $\tau=\text{const}$ and $c_0=\text{const}$. From these curves, as well as from (12) and (15), it follows that $\nu^*=\text{const}$. Thus, the independence of the length of the chain ν^* from the stage of the reaction can be considered as a fundamental law of the development of (isothermal) chain reactions.

In the case of rapidly proceeding reactions, as a rule, the isothermal conditions are infringed. Such is the case, for instance, in the experiments of Kovalsky on the oxidation of hydrogen. For comparison with the theory the thermal autoacceleration of the reaction should be taken into account in this case.

From (15) and (3) after differentiating we find

$$\frac{d\alpha}{dt}=\frac{c_\alpha}{\tau} \tag{16}$$

where

$$c_\alpha=\frac{(\alpha+c_0)^2(1-\alpha)}{\alpha+c_0-(1-\alpha)\ln(1-\alpha)}$$

and further

$$\varepsilon=\varepsilon_0\left(\alpha+\frac{d\alpha}{dt}t\right) \tag{17}$$

where

$$\varepsilon_0=(x\delta-1)=\frac{\Delta\tau_0}{\tau}.$$

Attributing to α different meanings, we can find, by using (14), (16) and (17), the corresponding values for t, $\frac{dx}{dt}$ and ε and thus define the course of kinetical

curves. As an example Fig. 1 shows theoretical values for $\alpha(t)$ and $\varepsilon(t)$ [$\tau=9''$, $c_0=0.03$]. For sake of comparison we give there experimental data obtained by Kovalsky, Sadovnikov and Chirkov ([4]) for the products of the oxidation of methane. As it may be seen from the figure, the agreement is excellent.

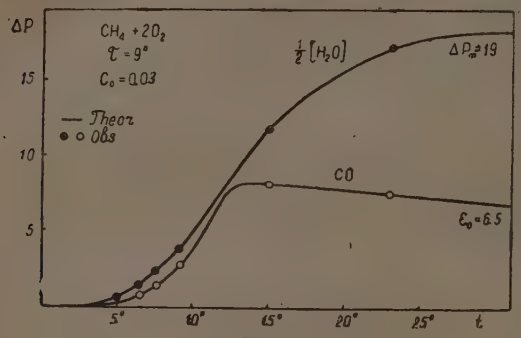

Fig. 1

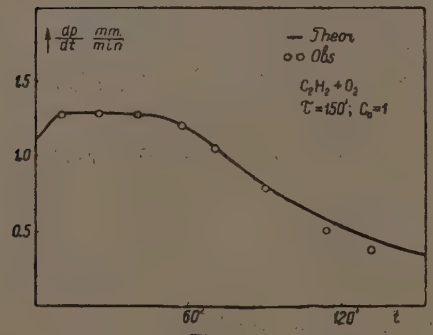

Fig. 2

Equations (11), (15) and (17) make it possible to prove all the main theses of the dynamics of chemical reactions as developed previously by the author, and to give the solutions of the fundamental problems of the theory of combustion, inflammation and explosions.

As to the formula for the rate of the reaction (16), with $c_0 \to 0$ as a limit it passes over into a formula expressing the effect of induction and autogenetic jump in the velocity, followed immediately by a rapid drop. Considerable interest belongs also to the case of large c_0.

In this case from (16) and (14) follows the existence of a phenomenon of quite a different type, viz. the effect of a practical constance of the rate of the reaction, beginning from $t=0$ to $t_a \approx \frac{1}{2}$ (by the order of magnitude), after which a rapid decrease in the velocity takes place.

This effect has actually been observed by many authors. In particular, it takes place in the remarkable results of the experiments by Kistjakowsky and Spense on the oxidation of acetylene (Fig. 2). This effect (catagenesis) is due to a superposition of two processes—catalysis and autogenesis, each of them giving maxima for the rate of inversion, the position of which does not coincide. In this way is obtained the curve of the type shown in Fig. 2. As a rule, between the curves of purely autogenetic and purely catalytic type there can be observed a scale of transitions which is actually observed in the immense experimental material available on the kinetics of chain reactions.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

PLANT PHYSIOLOGY

COMPARATIVE METHOD FOR THE EVALUATION OF THE SALT RESISTANCE OF COTTON PLANTS

By P. A. HENKEL and S. S. KOLOTOVA

(Communicated by A. N. Bach, Member of the Academy, 23. VIII. 1942)

A comparative method for evaluation of the salt resistance is described in this paper. So far it has been applied forthe sake of comparing control cotton plants with plants given a pre-sowing treatment with salt solutions, and also for the estimation of the salt resistance of cotton plants grown on soils with different contents of alkalies.

The strengthening of salt resistance by pre-sowing treatment was carried out by a method developed for wheat ([1]), but the concentrations of the salts used for the treatment of the seeds were different. In the case of cotton a 4% solution of the salt mixture was applied in the proportion recommended by Vant Hoff ([1-3]).

Germinated seeds of an American cotton, variety 8517, were treated for 1.5 hours by Vant Hoff's salt solution and then sown in the field on test-plots.

For determining the salt resistance we used as an indication the appearance of light spots and streaks on the leaves upon their poisoning with salts. In the literature we found data by Monfort ([4]) who noticed that on the immersion of leaves from different plants by their petioles in the salt solution at first the hydrolysis of starch set in, then the leaves began to loose their colour, and finally a complete destruction of chloroplasts took place. Morphologically one may point out the formation of typical yellow spots on leaves. It is of interest to remark that the leaves of halophytes in Montfort's experiments showed no similar signs of suffering from the excess of salt. Strogonov and Ostapenko ([5]), while subjecting cotton leaves to the action of extracts from soils of different alkalinity, likewise observed the appearance of light spots and streaks.

In the summer of 1940, when we were studying the effect of salts on the process of transpiration in the leaves cut off and immersed in salt solutions, we observed the formation of typical salt spots on cotton leaves. The intensity and the speed with which these spots turned up varied according to the alkali content of the soil on which they had grown. This observation suggested the idea of working out a comparative method for the estimation of salt resistance in cotton plants.

The method we are to propose here may briefly be described as follows. The leaves from a definite node are severed and immersed with their petioles into flasks or glasses filled with solutions of various salts, 0.1 molar concentration. Previous to the immersion the bases of the petioles are cut under water to attain equal length, and the leaves are placed for 30—50 min. under conditions of intense transpiration in direct sunlight. 10—15 min. later small isolated spots usually appear on the leaves and gradually coalesce, so that by the end of the experiments continuous yellowish-white streaks are formed. The spots appear first in the parenchyma at some distance from the main vein; by and by they spread to the leaf periphery.

The formation of salt spots is most conspicuous in the solutions of sodium sulphate and calcium chloride,but it is also observed in the solutions of other salts, such as sodium chloride, potassium chloride, magnesium chloride, magnesium sul-

phate and potassium sulphate, as well as in a balanced solution of Vant Hoff. The character of the spots (their size) varies within a narrow range, according to the salt applied. In the solutions of calcium chloride and sodium sulphate the spots stand out with especial clearness, apparently, because the dehydrating property of calcium and of the anion of sulphuric acid are expressed with particular strength.

The reason why the experiment should be carried on under conditions of intense transpiration is that otherwise it would last too long (two or three hours) and give less clear results on account of the falling of leaves. A small proportion of leaves wither even under conditions of intense transpiration on direct sunlight. Naturally enough, such leaves must be discarded from the experiment, for being not overstored with salts they will form no salt spots. Aside from test plants immersed in the salt solution, part of the leaves were placed in flasks with distilled water to serve as controls. The degree of injury caused by the salts was expressed in terms of a five-point scale: 0, no salt spots; I, weak spots; II, intense spots; III, feeble streaks; IV, intense streaks. The arabic subscript indicated the number of leaves showing the respective degree of affection. There were 3 leaves in every set. Of the numerous tests made by us the results of only a few are summarized in the tables.

We shall first turn to the experiments made with the leaves from soils of different alkalinity. As is seen from Table 1, the most salt-resistant plants are from a strongly alkaline plot. In these plants the formation of salt spots sets in considerably later than in plants from a weakly alkaline plot.

Table 1

Affection of Cotton Leaves in a 0.1 Molar Solution of Na_2SO_4

Alkalinity of soil	Node	Duration of exper. in min.	Affection
Weak	17	30	III_3
Moderate	17	30	II_2, III_1
Strong	17	30	I_1, II_2
	15	40	IV_3
Weak	18	40	III_3
	22	40	II_{12}, 0

Table 2

Affection of Leaves in Control and Test Plants by a 0.1 Molar Solution of Na_2SO_4

Set	Node	Duration of exper. in min.	Affection
Control	16	25	III_3
Test	16	25	I_3
Control	16	25	IV_2, II_1
Test	16	25	II_3

The results obtained have placed beyond doubt the fact that the plants from strongly alkaline soils are better adapted to the toxic effect of salts. The threshold of toxicity is higher for these plants, because they have adapted themselves in the process of development to the salts present in strongly alkaline soils.

This process of adaptation to salts has been reported more than once by several different authors ([8], [4], [7]) in a number of halophytes and glycophytes, and the results obtained by us have furnished additional confirmation to this point of view. From Table 1 it may be inferred that the resistance of cotton leaves varies with the age: the leaves from the upper nodes which [see Krenke ([6])] are the youngest with regard to their particular, and the oldest with regard to their general age, are distinguishable by a stronger resistance, compared with the leaves lower down the stem.

The next set of tests was devoted to comparative testing of leaves from control cotton plants and from plants given pre-sowing treatment in salt solutions.

It may be seen from Table 2 that the leaves of the test plants showed stronger resistance to salts. It is interesting to note that the difference between test plants and controls did not vanish during the start of ripening. In other words, the effect of pre-sowing treatment involved the whole vegetation period, as regards the strengthening of salt resistance.

To produce a more decisive evidence that the threshold of the toxical action of salts is increased by pre-sowing treatment with salt solutions, we made a direct estimation of the content of chlorine and SO_3 in the dry matter of the leaves taken for the experiment. The chlorine was determined by titration with silver

nitrate; SO_3, by titration in the presence of sodium rhodizonate. It was found (Table 3) that with equal strength of affection, which in the test plants appeared later, the content of chlorine and SO_3 (in the leaves from the 20th node) was considerably higher in the experimental plants than in controls.

Table 3

Sets	Mgs per 1 g of dry matter			
	Water	NaCl	$CaCl_2$	$MgCl_2$
Control . .	0.27	6.8	10.3	13.5
Treated . .	0.13	10.1	14.0	17.5

Similar picture was observed by us on analysing the content of chlorine ion in the plant juice of control and experimental plants*.

Contrary to the previous experiment, this time the leaves were not kept in the different chlorine salt solutions until they showed an equal degree of affection, but they were kept there during an equal length of time measured in minutes. From Table 4 (16th node, duration of experiment 30 min.) it may be seen that the test cotton plants had more chlorine in 1 cm³ of the juice and their affection was less.

Table 4

Sets	0.1 mole of NaCl		0.1 mole of $MgCl_2$		0.1 mole of KCl	
	Chlorine in mg	Affection	Chlorine in mg	Affection	Chlorine in mg	Affection
Control	2.34	III_1, II_1	3.05	IV_2	2.34	I_3
Test	2.98	I_2	4.51	II_2	3.66	I_1, 0_2

A higher content of SO_4 ion ** in the juice and a somewhat weaker affection of leaves was also noticed by us in the cotton plant from more alkaline plots.

Table 5

Content of SO_4 Anion in the Juice of Cotton Leaves of the 16th Node Grown on Soils of Different Alkalinity (duration of experiment 30 min.; leaves kept in a 0.1 molar solution of Na_2SO_4)

Alkalinity of soil	Contents in mg	Affection
Weak	1.7	IV_3
Moderate	2.4	IV_2, III_1

This is obvious from the data referred to in Table 5. The data of analyses speak decisively for the possibility of using the method proposed for a comparative estimation of salt resistance in cotton, for in all cases there was observed an increase of the threshold for the toxic effect of salts upon plant tissues.

The method is likely to be of value for a rough estimation of salt resistance of different varieties of the American and Egyptian cotton. It may prove useful in the case of other cottons as well. At any rate similar salts spots were observed to appear on the leaves of alfalfa and Indian corn kept in salt solutions.

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Received
11.VIII.1942.

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* The juice was expressed from leaves killed by steam in Koch's boiler.
** The estimation was made by the benzidine method.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

PLANT PHYSIOLOGY

ACCELERATION OF GROWTH IN SUCKERS OF *HALOXYLON APHYLLUM*

By V. LEONTIEV

(Communicated by B. A. Keller, Member of the Academy, 17. VI. 1942)

Under conditions prevailing in the Kara-Kumy desert the sowing of *Haloxylon*—and also the self-propagation of this plant by occasional dropping of seeds—is far from being successful every year (there may be intervals of 8 to 10 years). On the other hand, failures experienced with cuttings of *Haloxylon* and the fact that their planting is a costly and cumbersome operation, compelled the author to look for new, more effective methods for the restoration of *Haloxylon* areas. Very promisng results have been obtained by using the surculose renewal of growth and caring for the young plants thus produced.

The suckering power of *Haloxylon* has been known for a long time ([1–4]). But until recently the information concerning the development of *Haloxylon* suckers was very scant. One thing only was regarded as certain. Namely, that the stumps of both *Haloxylon aphyllum* and *Haloxylon persicum* produce strongly tillering surculose growth. Of these two plants the first is of greater interest for sylviculture. So we shall center our attention about the possibility of its propagation by suckers in Turkmenia. As a result of this author's investigation it has been established that the stumps of *Haloxylon aphyllum* measuring 24 cm or less in diameter nearly always give surculose growth when cut above the root crown (regardless of the way of felling and of the height of the stump). Stumps 32 cm in diameter or more give no surculose growth with rare exceptions, and those between 25 and 32 cm sometimes produce suckers and sometimes do not.

The surculigenous ability is not so much influenced by the diameter of the stump, as by the age of the felled shrub. But no reliable method for determining the age of *Haloxylon* shrubs being available, I was compelled to use the stump diameter as a basis*.

With reference to age the shrubs of *Haloxylon* may be classed into 5 groups: young (1—10-year-old), medium-aged (10—18), ripening (18—23), ripe (23—30), over-ripened (above 30). The young and the medium-aged always produce suckers; the over-ripened never does, while the ripening and ripe sometimes do and sometimes not.

The time of felling has a certain influence on the development of the suckers. On stumps left previous to vegetation or at the beginning of that period the shoots have more time for development than if the felling is made in summer. From sound stumps of *Haloxylon aphyllum* the surculose shoots appear out of dormant buds in great numbers. In all cases the suckers of the two species of *Haloxylon* tiller strongly, make intense growth, but by the end of summer turn brown and their growth comes to an end before the vegetation period of seed-grown *Haloxylon* shrubs is

* The determination of age from branching according to Artzikhovsky is liable to give very inaccurate estimates, and from their general habitus may be only referred to a certain age group.

over. During the second year growth is slowed down, but tillering is still strong. From this, sylviculturists have concluded that no use can be made of surculose growth for the production of wood (²). To my opinion, the fact that the suckers turn brown in summer and cease growing too early may be explained by the water balance being disturbed in the plant. The roots send no sufficient moisture to the vigorously growing and strongly transpiring juicy shoots; all the more so because they themselves during the first period receive no food from the crown. Thus, 8- and 10-year-old summer specimens of surculose growth under average conditions (in *Haloxylonetum fructicoso-herbosum* growing among underwood and different types of grass) attain a height of 2 m, 2 m 13 cm at most, while tillering strongly and giving very little new growth during the last years. Seedlings of *Haloxylon aphyllum*, raised from seeds simultaneously with the suckers in a biologically similar environment, had attained in the same space of time a height of 2—3.5 m, looked tree-like and gave a twice higher new growth during the last year.

Both the seedlings and suckers of *Haloxylon* make more rapid growth than is believed generally (Paletsky, Novikov). However, under average and even optimal conditions of growth the adopted period between fellings is 30 years with quite uncertain renewal dates ranging from 2 to 10 years.

In January 1939 I started a new experiment on surculose renewal of growth. On a band test-plot 10 m broad, 2.5 km long (2.5 ha), stretching across the black (*aphyllum*) and other kinds of *Haloxylon*, all the trees and shrubs were broken mostly after making a cut with an axe. The suckers that appeared in spring rapidly covered the whole area by nearly continuous brush growth. In April, after measuring the longest shoots, only the most vigorous one was left on every stump in order to reduce the deficiency of moisture. All the other shoots were broken off. This caused a marked acceleration of growth in the shoots left, but a few days later unexpanded buds on the stems, which had previously been in a state of dormancy , sent forth new abundant suckers. These were broken off again at the beginning of June. By this time the shoots left behind earlier were already 40—45 cm to 1 m high. Lateral branches that had appeared on their lower parts were removed, and this was repeated in August. Even in this first year of growth the shoots so cared for came to complete their vegetation simultaneously with the seedlings of the foregoing years instead of before, as had been observed to be the case with uncared for suckers, and their stems by this time were, moreover, higher and thicker. They went well through winter and dropped their assimilation shoots simultaneously with the adult specimens of *Haloxylon*.

On the second year the same procedure was made in May after measuring the remaining shoots. Now these shoots were already bearing small crowns having produced several (instead of two normal) branches in one and a half growing periods. By the end of their second growing period they reached a height of 2 to 2.5 m and measured up to 7 cm across.

By the end of 1941, i. e. in three years of vegetation, the surculose trees of *Haloxylon aphyllum* attained a height of 2.5—3 m and some were even more than 3 m high; the diameter of the stem amounted to 8 cm.

In an experiment with medium-aged and young shrubs of *Haloxylon* more than 90% of the stumps produced suckers. There was no subsequent falling among this growth.

If cared for, the suckers increase their rate of growth more than thrice (2.47 m in two years instead of 2.19 m in eight years) and continue so in the third year. On part of the test area all surculose shoots without a sole exception were regularly broken off during two years and in the spring of the third (1941). There resulted an avenue 4 m wide which toward the autumn of 1941 was covered throughout its length of 2.5 km by a new surculose growth, though a somewhat less abundant one. This demonstrates the strong vitality of free rooted *Haloxylon* and its high powers of repeated regeneration.

The caring-for described is extremely simple and requires no much labour, except subsequent breaking off, absorbing less work than the preceding one. The

refuse of broken off shoots can be turned to profitable account in the manufacture of soda and potash.

In that way tracks covered by medium-aged *Haloxylon* shrubs may be exploited. The method of felling does not affect the surculose power, if the stumps left include the root crown.

The period between fellings can be reduced from 30 years to 10. Ripening as well as medium-aged shrubs of *Haloxylon aphyllum* may be so exploited, if the rapid renewal of growth is ensured by the subsequent care. It is well to leave 10 such producers per ha which will ensure the dispersal of seeds, should the *Haloxylon* shrubs loose their surculose ability as a result of reiterated fellings of sucker plantations. Over-ripe trees must be lifted up with their roots as they give no surculose growth.

This new method for the acceleration of growth in *Haloxylon* affords the possibility to produce valuable fuel wood in as short a space of time as with the rapidly growing shrubs of cherkez and kandym.

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Received
17.VI.1942.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

PHYSIOLOGY

NEW DATA ON THE EFFECT OF BENZEDRINE AND PERVITINE UPON THE NERVOUS SYSTEM

By CH. S. KOSHTOJANZ, Corresponding Member of the Academy, and R. L. MITROPOLITANSKAYA

Now, under the conditions of war, physiological stimulators, including such synthetic preparations as benzedrine (1-phenyl, 2-aminopropane) and pervitine (or methylbenzedrine: 1-phenyl, 2-methyl-aminopropane), have attracted particular attention of physicians and physiologists of many cliniques and laboratories all over the world. The action of these substances is to reduce the sensation of weariness, to increase physical and mental working capacity and, moreover, sharply to suppress the requirement of sleep. Owing to these qualities, they are used in modern armies as stimulants and as a remedy against fatigue. The range of cases of sharp disturbances of the function of the nervous system when the use of this substance is indicated becomes wider and wider. As may be seen from special literature, benzedrine is also applied in the English aviation as a means serving to increase the resistance of the pilot's organism to altitudinal anoxaemia.

General questions of the chemical composition of benzedrine and pervitine, of their physiological effect and the peculiarities in the mode of application as compared to that of the related preparations of the group of adrenaline and ephedrine, have already been treated by one of the authors ([1]). The present note contains the results of new experimental work that has been carried out with benzedrine synthetized in the laboratory for heterocyclic compounds at the Institute of Organic Chemistry of the Academy of Sciences of the USSR, and with pervitine, synthetized in the Kharkov Institute for Experimental Endocrinology.

I. Effect of Pervitine upon the Sleep of Hibernating Animals. Benzedrine and pervitine have earlier been found to interrupt a sound sleep in man, including sleep induced by dormitives. These substances thus are antagonists of dormitives, and have received the appropriate name of «awaking amines». On the ground of the data referred to above we undertook to test the effect of benzedrine or pervitine on hibernating animals at the period of their profound winter-sleep. Our purpose was to find out whether these substances will be able to interrupt the state of profound sleep when applied to hibernating animals 1.5 months before the end of their winter-sleep.

The experiments were carried out at the town Frunze during January and February. As test-animals served hedgehogs and hamsters which have been kindly put at our disposal by Dr. A. D. Slonim, head of the Department of Physiology at the Kirghiz Medical Institute. In the course of these experiments we recorded with the help of an actograph the day-and-night motor activity of the animals under test, before and after the application of pervitine. As shown by these experiments, about an hour upon a single application of 5—10 mg of pervitine a hedgehog fast asleep awoke and remained in a state of unusual motor activity for 2—4 hours. After this the animal fell asleep again. A new dose of pervitine once more interrupted the sleep of the animal; during the period of time indicated above it developed again a high motor activity; after the effect of the drug was over, the sleep recommenced. Control experiments, involving the administration of sodium chloride and

sugar (the latter entered into the composition of the pervitine troches used by us), failed to produce the effect of awaking.

The experiments described will at the next season be supplemented by tests involving a regular administration of pervitine or benzedrine in order to bring about an irreversible break of the profound sleep of hibernating animals. However, the reversible influence brought about by single applications of pervitine suffices to give a clear idea of the awaking effect of stimulants of the type of benzedrine and pervitine. And this effect is such as to break profound sleep in hibernating warm-blooded animals.

II. On the Action of Benzedrine upon the Cholinesterase. The well-pronounced stimulating effect of benzedrine and pervitine upon the nervous system could not fail to give rise to investigations of the chemical mechanisms, by the way of which these substances affect the nerve cell. Mann and Quastel [2], the well-known Cambridge biochemists, dealing with the brain tissue, have recently reported that benzedrine produces a specific inhibitory effect upon the action of aminoxidase, thus delaying the formation of products inhibiting the chemical processes of respiration of the brain tissue (in particular, of certain aldehydes), and preserving thereby this tissue from a decrease in the intensity of oxidizing processes. It is this ability of benzedrine indirectly to enhance the intensity of oxidizing processes of the nerve cells that underlies the increase in the resistance of organisms to altitudinal anoxaemia.

As known from numerous modern studies, inclusive our own [3], an important part in chemical transformations of the nerve cells belongs to the enzyme cholinesterase. With the action of this enzyme are connected processes of synthesis and hydrolysis of acetylcholine (and of other esters of choline), which plays a definite rôle in the phenomena of functional activity of nerve cells. Considering all that had been said above, we undertook to test the effect of benzedrine upon the activity of the cholinesterase.

The experiments were carried out with the material generally made use of, viz. with the dorsal muscle of the leech. As a source of cholinesterase we made use of equine serum. The serum was used in the following dilutions: 1 : 10, 1 : 100, 1 : 1000, 1 : 10 000, 1 : 100 000. Benzedrine was tested in the following dilutions: 2 mg %, 4 mg %, 20 mg % and 40 mg %. Acethylcholine bromide was taken in a dilution 1 : 1 000 000. The application of acethylcholine solution invariably gave a typical effect of a sharp contraction of the dorsal muscle of the leech. In most of the experiments, a serum diluted 1 : 10 000, when left to stay 20 hours in a test-tube with a solution of acetylcholine, entirely eliminated the effect of acetylcholine, the latter having undergone hydrolysis under the influence of the serum's cholinesterase. The same experiments show that when benzedrine is added to the mixture of serum and acetylcholine, the hydrolysis of acetylcholine sets in with considerably lower dilutions of the serum, say, with 1 : 1000. The concentration of the benzedrine solution was found to play no part in this phenomenon. All the solutions of benzedrine tested by us showed on the average a tenfold inhibition of the activity of the cholinesterase. From the above said it may be concluded:

1. The effect of pervitine and methylbenzedrine is to interrupt a profound sleep of hibernating animals (hedgehog, hamster) during the winter season. An objective record taken with an actograph shows that as a result of a single injection of the above substances hibernating animals entirely awake from their sleep and display an energetic motor activity.

2. Benzedrine produces an inhibiting effect upon the enzyme cholinesterase.

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Received
4. XII.1942.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XXXIX, № 5

PARASITOLOGY

PEDICULOIDES VENTRICOSUS IN IRAN

By E. N. PAVLOVSKY, Member of the Academy

A peculiar affection of the cutaneous coverings was observed in autumn 1941 among a group of persons who came to visit Gorgan (Astrabad) for the first time. In the subsequent it was found that all of them had to deal in some way or other with straw of rice, wheat or barley, used for stuffing mattresses to sleep upon. In November and December 1941 a similar disease was recorded at various provinces of Gorgan and Masenderan among persons who had been working to load straw in cars and motor-cars. The disease was found to spread also among persons who used for their bedding mattresses stuffed with this straw.

Special interest deserve some cases of the disease, reported from the vicinity of Gorgan, where it was observed among hunters who, when chasing, concealed themeselves in heaps of straw. There was also a case when a man who had nothing to do with straw, fell ill a month after he had purchased boots and gamashes from a person employed to load straw.

The disease was characterized by an abundant eruption consisting of isolated or multiple exanthemata or pustules, which sometimes coalesced, covering the whole body surface, except the palms and the soles. The eruption was accompanied by a most irritating and sometimes unbearable itch, that increased towards the evening and made the patients scratch themselves. According to the intensity of scratching, there was excoriation of the epidermis accompanied by exudation of lymph or blood, and sometimes attended with an infection with foreign pyogenic bacteria.

One of the patients (a chauffeur, who had been loading straw) showed especially grave lesions of the face and neck; owing to eruptions and the itch, the face was so swallen and oedematic that the eyelids could not be set apart unless with the help of fingers.

In those of the patients who showed extensive lesions of the skin, the disease was accompanied by chill, elevation of temperature, headache, abatement. In women the disease assumed a much graver course than in men.

The duration of the disease varied as to the case. If the straw mattresses and pillows were taken away and the patients given clean sleeping things and linen, the itch became less intense and ceased altogether towards the third or the fourth day. The eruption became paler and disappeared by the fifth or sixth day. At the places of the pustules and exanthemata there remained small browny pigment spots. Occasionally the disease lasted as long as two or even four weeks. The treatment consisted in the application of neutral pastes or itch assuaging solutions.

The aetiology of this disease failed to be elucidated by local medical officers. The only thing made sure was that all these who fell victim to the disease had been in contact with straw, either as loaders, or using it for their pillows and mattresses. The information available was supplied by Dr. V. M. Telzov who personally attended to a number of cases.

I became aware of the cases described above in May 1942, upon my arrival at Gorgan at the head of my epidemiological and parasitological staff. I at once felt suspicious that the tick *Pediculoides ventricosus* might be involved in the affair; I ordered therefore several dots of saman (local name for cut straw), and under the microscope the very first samples were found to be infested with non-pregnant females of *Pediculoides ventricosus*. The cause of the disease has thus been made evident.

As well known, *Pediculoides ventricosus* is a parasite which devours larvae of noxious insects inhabiting stalks of various grain crops. In case they are short of this kind of food, however, the ticks leave their abode in straw or grain, and attack men who happen to handle these materials. In men they provoke a disease which develops rapidly and causes most annoying sensations; and this was the case in Iran. The disease may assume the character of an outbreak which suddenly declares itself among workmen having to do with grain, straw and so on.

Pediculoides ventricosus and the disease caused by it have been reported from a number of southern localities, viz. from Bulgaria, Italy, Southern France, etc. These data are in good agreement with the fact that *Pediculoides ventricosus* has been found in Iran, where it is undoubtedly of much wider occurrence than supposed heretofore.

In order to protect men from the painful disease caused by *Pediculoides ventricosus*, the straw designed to serve for stuffing mattresses and pillows should previously be exposed to sunlight during two days; it must be spread as a thin layer over a firm plot of bare land and turned over from time to time.

2nd Iranian Epidemiological and
Parasitological Expedition
Gorgan.

Received
9. XI. 1942.

Prix $ 0.50